it is necessary to perform in the future further analysis of nonstationary conditions at the wall.

NOTATION

ψ, the wave function; y, radial distance from the tube axis; t, time; U, absolute value of the translational velocity; ρ, density (incompressible flow); a and b, wave amplitude and phase; ω, fluctuation frequency; R, tube radius; τ, shear stress; τ_0, shear stress at the wall; μ, viscosity coefficient; h, a "quantum" parameter, related to the circulation of large vortices; λ, hydraulic resistance coefficient; Re, Reynolds number.

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MATHEMATICAL BOUNDARY-LAYER MODEL FOR A WIDE RANGE OF TURBULENT REYNOLDS NUMBERS

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UDC 532.517.4

Based on the e—ε turbulence model, a boundary-layer system of equations is proposed, describing the laminar, transition, and turbulent flow regimes.

Analysis of contemporary turbulence models [1] shows that the most promising models for describing turbulent transfer processes in boundary layers are those in which the fluctuating flow characteristics are determined as a result of simultaneous solution of the equations of turbulence intensity e and dissipation ε. For the development of turbulent flows with relatively large turbulent Reynolds numbers ($R_T = e^2/(\nu\varepsilon) > 10^3$) models of this type have been developed in detail [2] having many practical applications. In describing flows with small R_T e—ε models were first used in [3]. In this case additional corrective terms and closure functions were introduced in the equations of turbulence intensity and dissipation, but justifying several of their assumptions seemed to raise doubts [1]. Thus, for example, introduction of the additional term $-2\mu(\partial\sqrt{e}/\partial y)^2$ in the right-hand side of the e equation, due to nonvanishing of dissipation at the wall, destroys the total balance and leads to lowering of the solution stability for increasing step sizes in the longitudinal direction. There is no physical justification for the further term in the dissipation equation $2\mu_T\nu(\partial^2U/\partial y^2)^2$, which seems to affect substantially the solution results in the direct neighborhood of the streamline surface, where the gradients of the flow parameters are particularly significant.

Despite the fact that by means of the Jones—Launder model [3] it seems possible to calculate several important special cases of boundary-layer flow, such as flow with acceleration,

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the problem of creating universal e—ε models is far from completion. The available models do not allow one to obtain stable boundary-layer solutions in the transition from the laminar to the turbulent flow regimes, and even in the case of calculating "laminarization" effects of accelerated flows their accuracy remains low.

A method is suggested below of dissipation closure equations for the case of relatively small Reynolds numbers. All closure coefficients are determined as a result of processing experimental data, and to account for the flow features directly near the streamline surface one additional closure function is introduced into the dissipation equation. As a result of a numerical experiment it has been established that in this case the problem of assigning boundary conditions for the dissipation equation practically does not affect the results of calculating mean flow parameters. Unlike the Jones—Launder model, the suggested model is simpler and contains fewer closure relations.

The Dissipation Equation. Due to the complexity of closure the dissipation equation has so far not been obtained in general form. For the cases of locally equilibrium turbulence or flow with relatively large turbulence Reynolds numbers [4] the expression for dissipation is substantially simplified, and the equation can be obtained from the Navier—Stokes equation [5]. The terms of the equation, containing velocity correlations of second and third orders, are expressed by dimensionality considerations and available experimental data in terms of some mean flow parameters, turbulence intensity, dissipation, and closure coefficients. For a two-dimensional stationary flow the dissipation equation is written in the form [2]

$$\rho U \frac{\partial \varepsilon}{\partial x} + \rho V \frac{\partial \varepsilon}{\partial y} = \frac{\partial}{\partial y}\left[\left(\mu + \frac{\mu_T}{\sigma_\varepsilon}\right)\frac{\partial \varepsilon}{\partial y}\right] + C_1 \mu_T \frac{\varepsilon}{e}\left(\frac{\partial U}{\partial y}\right)^2 - C_2 \rho \frac{\varepsilon^2}{e}. \tag{1}$$

The turbulent transfer coefficient μ_T is defined by the equation

$$\mu_T = C_\mu \mu R_T. \tag{2}$$

According to the first Kolmogorov hypothesis [6], the coefficients of Eq. (1) and expression (2) for flows with relatively large R_T are constant and are determined on the basis of experimental data, as was shown, for example, in [2]. Very near the streamline surface the local turbulent Reynolds number is small, and the viscous friction forces become commensurate with the friction forces due to turbulent liquid motions. In this case the coefficients in Eq. (1) and expression (2) are functions of the turbulent Reynolds number.

The specific functional dependences for the closure coefficients C_1, C_2, C_μ, and σ_ε in the present study were obtained on the basis of processing experimental data on measurements of turbulence structure in tubes [7], boundary layers [8], boundary layers with positive pressure gradients [9], etc. The papers mentioned contain information on the profile distributions of the mean velocity U, the fluctuating velocity components u_i', and the turbulent tangent stress $\langle u'v' \rangle$ as a function of the transverse coordinate y. Introducing these original data into the turbulence intensity equation, which is simplified by the assumption of small convective terms in comparison with the remaining ones

$$\frac{d}{dy}\left[\left(\nu - \frac{\langle u'v' \rangle}{dU/dy}\right)\frac{de}{dy}\right] - \langle u'v' \rangle \frac{dU}{dy} = \varepsilon, \tag{3}$$

we obtain a system of equations in the unknown function ε(y), selected in the form of a third-order polynomial $\varepsilon(y) = a + by + cy^2 + dy^3$. As a result of solving the equations of this system by the least-squares method for each series of experiments the dissipation value is determined as a function of the coordinate y. Following that, the value of the coefficient C_μ in Eq. (2) is calculated by the relation

$$C_\mu = -\frac{\langle u'v' \rangle \varepsilon}{dU/dy \cdot e^2}. \tag{4}$$

As follows from the data presented in Fig. 1 (points), for $R_T > 10^3$ the coefficient C_μ can be assumed to be a constant quantity, equal to 0.09 with an accuracy of ±10%. With decreasing number R_T one observes a gradual lowering of the coefficient, which can be described by the exponential dependence

$$C_\mu = 0.095\left[-\exp(-2.5) + \exp\left(-\frac{125}{50 + R_T}\right)\right] \tag{5}$$

(the curve in Fig. 1).

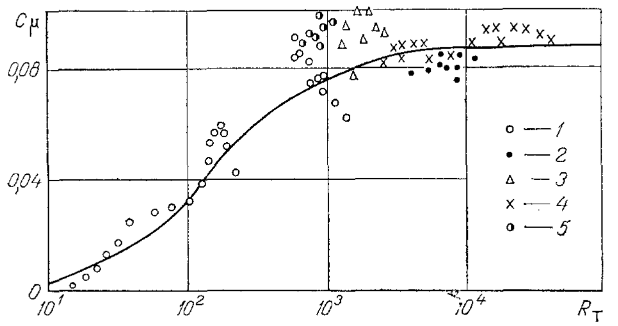

Fig. 1

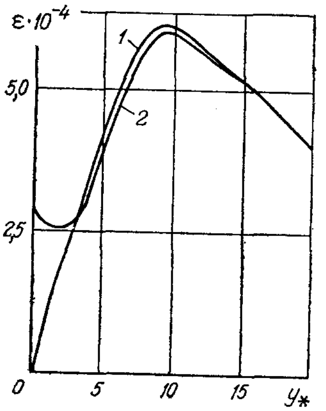

Fig. 2

Fig. 1. The coefficient C_μ (5) as a function of turbulent Reynolds number $R_T = e^2/(\nu\varepsilon)$ (points are results of processing experimental data, as obtained in the present study, 1) by the data of [7], Re = $5 \cdot 10^4$; 2) [7], Re = $50 \cdot 10^4$; 3) [8]; 4) [9]; 5) [10]).

Fig. 2. Calculation of the dissipation profile by the suggested model near the wall for different boundary conditions for the dissipation $\varepsilon \cdot 10^{-4}$, m^2/sec^3.

By processing experimental data it has been established that for any turbulence Reynolds numbers the coefficients C_1 and σ_ε are constant quantities, equal, respectively, to 1.65 and 1.3. The shape of the functional dependence of C_2 is determined as a result of processing the experimental data of [10] and can be represented in the form

$$C_2 = 2\left[1 - 0.3\exp(-R_T^2)\right]. \tag{6}$$

The data obtained for the closure coefficients were in total agreement with the results of [3]. However, the e—ε model closed only by means of these coefficients does not make it possible to calculate the boundary layer parameters near the streamline surface. The profiles of mean and fluctuating flow characteristics do not correspond in this case with the experimental data. Moreover, the calculation leads to negative values of the turbulence intensity and dissipation for $y_* < 3$. The reason for the deviation is that the conditions of locally isotropic equilibrium, under which assumption Eq. (1) was derived, are violated in the boundary-layer regions, where the turbulent Reynolds numbers are small. It has not been possible to account for these variations in turbulence structure only by means of the suggested closure coefficients.

The quantity ε can be considered as the isotropic part of the total dissipation D. It has been established experimentally [11] that the total dissipation does not vanish at the wall. Account of this effect can be either assigned by corresponding boundary conditions for Eq. (1) at the wall [for $y = 0$ $\varepsilon = 2\nu(\partial\sqrt{e}/\partial y)^2$ or $\partial\varepsilon/\partial y = 0$], or by introducing the additional term $-2\mu(\partial\sqrt{e}/\partial y)^2$ in the right-hand side of the turbulence intensity equation, as was suggested in [3]. However, the use of nonvanishing boundary conditions for the ε equation in the form (1) is impossible. For $\varepsilon_W \neq 0$ the last term of the dissipation equation $C_2\rho\varepsilon^2/e$ in the wall region loses its physical meaning (for $y \to 0$ it tends to infinity, since $e_W = 0$). The introduction of the same additional term in the turbulence intensity equation changes the e equation itself and affects the solution results; therefore it was suggested in [12] to avoid this method. In this case the dissipation equation itself is modified, in whose last term one introduces an additional function, restricting the extent of dissipation decrease immediately near the wall; it is written in the form

$$C_2\rho\varepsilon\left[\varepsilon - 2\nu(\partial\sqrt{e}/\partial y)^2\right]/e, \tag{7}$$

while the boundary condition for ε at the wall is written in the form $\partial\varepsilon/\partial y = 0$. Truly, a similar modification affects the solution results not only in the boundary-layer zone, but also in the external boundary-layer regions, where the turbulence intensity gradients can be large, particularly in the transition flow regimes.

Despite the difference in shape between the turbulence intensity and dissipation equations, the calculations of the U and e profiles by the models of [3] and [12] coincide, as follows from the data provided in these studies. In this case, for correct description of the experimental e profiles in the boundary layer ($y_* \sim 15$) it is necessary to introduce into the dissipation equation one additional correction term $2\mu_T\nu(\partial^2 U/\partial y^2)^2$, for which no physical justification is given.

In the boundary-layer model suggested in the present study, the turbulence intensity and dissipation equations were included without introducing additional terms. To account for features of turbulent transfer in the boundary-layer flow region, where turbulence has an anisotropic nature, one introduces in the last term of Eq. (1) a correction function f. The purpose of introducing it is to restrict the growth of the dissipation degradation term near the wall. The variation region is from 0 to 1. The specific shape of the function f is determined from the conditions guaranteeing best agreement of the mean velocity and turbulence intensity profiles with the experimental data [7-10]. The ε profiles calculated in the present study and the experimental data from the studies mentioned on the U, u', v', w', <u'v'> profiles are substituted into the dissipation equation (the convective terms are also discarded)

$$\frac{d}{dy}\left[\left(\nu-\frac{\langle u'v'\rangle}{\sigma_\varepsilon dU/dy}\right)\frac{d\varepsilon}{dy}\right]-C_1\langle u'v'\rangle\frac{\varepsilon}{e}\left(\frac{dU}{dy}\right)=C_2 f\varepsilon^2/e, \tag{8}$$

as a result of whose solution we find $[(f=\varphi(y_*)]$:

$$f=-\exp(-10)+\exp\left(-\frac{250}{25+y_*^3}\right). \tag{9}$$

The y_* dependence of the function f was selected in such a manner that the dominant modification factor be the distance from the wall. We could not establish the dependence of f on R_T due to the substantial spread of the data obtained as a result of processing the various experiments.

To estimate the effect of boundary conditions for dissipation on the calculation results concerning the suggested turbulence model we carried out a numerical experiment. It was established that for the selected shape of the turbulence intensity and dissipation equations, as well as the closure relations, the shape of boundary conditions for ε at the wall does not seem to affect the velocity and turbulence intensity profiles obtained by the calculation. For the sake of comparison Fig. 2 shows the dissipation profiles near the streamline surface, calculated for the two boundary conditions $\varepsilon_W = 0$ (curve 1) and $\varepsilon_W = 2\nu(\partial\sqrt{e}/\partial y)^2$ (curve 2). It is seen that a difference in calculation results is observed only near the wall $y_* < 3$. Taking into account that in this region the turbulent exhange coefficient μ_T is negligibly small in comparison with the dynamic viscosity value μ, while the dissipation effect at the mean flow characteristic is mainly realized precisely in its terms, the absence of a substantial effect of boundary conditions for ε on the calculation results becomes understandable.

The choice of a boundary condition for the dissipation in the form $\varepsilon_W = 0$, but not $\varepsilon_W = 2\nu(\partial\sqrt{e}/\partial y)^2$ or $(\partial\varepsilon/\partial y)_W = 0$, is due to the fact that this homogeneous boundary condition guarantees highest solution stability for increasing difference steps of the grid.

Mathematical Model. Taking into account the dissipation equation obtained, the system of equations of the suggested turbulent boundary-layer model, together with the closure relations for flow of an incompressible liquid, is written in the following form:

$$\begin{gathered}
\frac{\partial\rho U}{\partial x}+\frac{\partial\rho V}{\partial y}=0,\\
\rho U\frac{\partial U}{\partial x}+\rho V\frac{\partial U}{\partial y}=\frac{\partial}{\partial y}\left[(\mu+\mu_T)\frac{\partial U}{\partial y}\right]-\frac{dp}{dx},\\
\rho U\frac{\partial e}{\partial x}+\rho V\frac{\partial e}{\partial y}=\frac{\partial}{\partial y}\left[(\mu+\mu_T)\frac{\partial e}{\partial y}\right]+\mu_T\left(\frac{\partial U}{\partial y}\right)^2-\rho\varepsilon,\\
\rho U\frac{\partial\varepsilon}{\partial x}+\rho V\frac{\partial\varepsilon}{\partial y}=\frac{\partial}{\partial y}\left[\left(\mu+\frac{\mu_T}{\sigma_\varepsilon}\right)\frac{\partial\varepsilon}{\partial y}\right]+C_1\mu_T\frac{\varepsilon}{e}\left(\frac{\partial U}{\partial y}\right)^2-C_2 f\frac{\rho\varepsilon^2}{e},
\end{gathered} \tag{10}$$

with closure relations

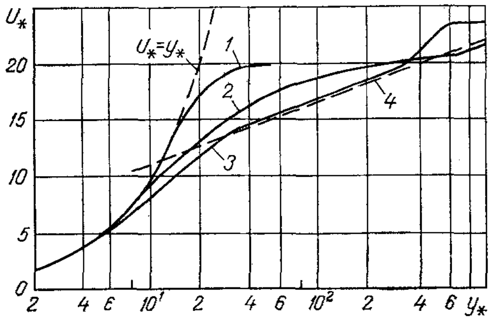

Fig. 3

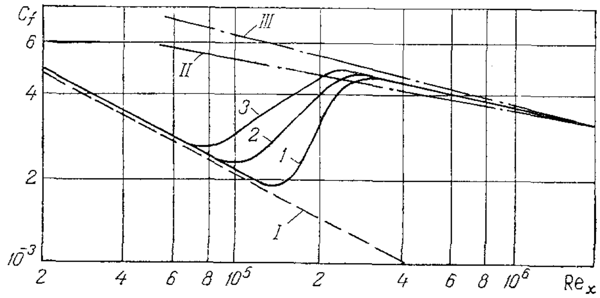

Fig. 4

Fig. 3. Velocity profile distribution in the transition region from laminar to turbulent flow (э = 0.5%): 1) $Re_x = 10^5$; 2) $2\cdot 10^5$; 3) $4\cdot 10^5$; 4) $u_* = 5.5 \log y_* + 5.45$.

Fig. 4. Results of calculating the local friction coefficient in the zone of the transition flow regime for varying turbulence intensity of the incoming flow: 1) э = 0.5%; 2) 1; 3) 2%; I) $C_f = 0.664/\sqrt{Re_x}$; II) $0.0263\ Re_x^{-1/7}$; III) $0.0592\ Re_x^{-0.2}$.

$$\mu_T = C_\mu \mu R_T,\quad C_1 = 1.65,\quad \sigma_\varepsilon = 1.3,\quad C_2 = 2[1 - 0.3\exp(-R_T^2)],$$
$$C_\mu = 0{,}095\left[-\exp(-2.5) + \exp\left(-\frac{125}{50 + R_T}\right)\right], \tag{11}$$
$$f = -\exp(-10) + \exp\left(-\frac{250}{25 + y_*^3}\right),$$

and boundary conditions

$$y = 0 \quad U_W = V_W = e_W = \varepsilon_W = 0,$$
$$y \to \infty \quad \frac{\partial U}{\partial y} = \frac{\partial e}{\partial y} = \frac{\partial \varepsilon}{\partial y} = 0. \tag{12}$$

The system of equations (10) was solved numerically by the finite-difference method. The solution through the boundary-layer width was performed by a single algorithm directly from the wall to the external flow region. The original equations and boundary conditions were approximated by a standard implicit finite-difference scheme. In what follows the calculations were carried out by using approximate numerical methods. To accelerate the calculation process and guarantee the required accuracy we introduced a modified coordinate system, providing "compression" of the transverse coordinate near the wall.

The conditions required to start the calculation were assigned by various methods. If the solution is realized from the laminar flow regime, then the theoretical profile for the Blasius solution for the velocity is assigned. Taking into account that in the laminar flow regime there exists fluctuating motion, though R_T is small, the initial turbulence intensity distribution was given in the form

$$e = 1.5\,\text{э}^2 U U_\infty, \tag{13}$$

so as to estimate the effect of the extent of turbulence of the running flow э on flow development in the boundary layer. The specific quantity э was selected from experimental data. The initial dissipation distribution was determined from the well-known Rotta equation [11]. For the case of the turbulent flow regime the initial conditions were assigned by known theoretical and experimental relations [13, 14].

According to the selected method of solving systems of differential equations of parabolic type, an algorithm was developed and programmed in the FORTRAN language. The width of the calculation zone was constant within one problem, and was determined in such a manner that at the end of the calculation it exceeded the thickness of the boundary layer by approximately five times. Also varied were the number of points across the flow and the grid condensation parameter in the direct neighborhood of the wall. A satisfactory calculation accuracy was achieved for 40 points across the computational grid for the nongradient flow regimes. In this case at least 50% of all points were located in the region $y_* < 50$, and three points — within the laminar sublayer.

Calculation Results. To estimate the validity of the closure relations and of the dissipation equations obtained we calculated velocity profiles in the boundary layer for the various flow regimes. A solution was found, starting at some point of a planar plate in which $Re_x \sim 10^4$, while the relative extent of turbulence of the running flow did not exceed 0.5%. Under these conditions the boundary layer is laminar and the calculated velocity profile coincides with the theoretical Blasius profile. Leaving the turbulent flow regime, the calculated mean velocity profile is totally rearranged and coincides quite well with a logarithmic distribution law (Fig. 3). Curves 1-3 correspond to Reynolds numbers $Re_x = 10^5, 2 \cdot 10^5, 4 \cdot 10^5$, and show how an increase in turbulent shear stress in the transition regime from laminar to turbulent flow deforms the mean velocity profile. In this case a laminar sublayer region with a linear velocity distribution $U_* = y_*$ is formed directly near the wall.

The laminar, transition, and turbulent solution regions of the system of equations of the given mathematical model can be simulated by the dependence of the local friction coefficient C_f on the Reynolds number Re_x (Fig. 4). The dashed curve corresponds to the dependence $C_f = 0.664/\sqrt{Re_x}$ from the theory of a laminar boundary layer, while the two upper dash–dot curves are the theoretical dependences of C_f for the turbulent flow regime, obtained from various initial references. The solutions obtained in the present study are shown by solid lines. They are in satisfactory agreement with available data. The transition region, obtained by the calculation, is determined by the extent of turbulence intensity of the incoming flow. The transition is shown in Fig. 4 for three different values. The larger the relative turbulence intensity, the earlier the transition starts.

All data calculated by the mathematical boundary-layer model suggested in this study agree well with data of theoretical and experimental studies, which confirms the validity of the closure relations obtained in the present study, particularly the correctness of selecting the homogeneous boundary condition for the dissipation equation.

NOTATION

x, coordinate along the flow; y, coordinate across the flow; U, velocity component along the x-axis; V, velocity component along the x-axis; p, pressure; u', v', w', or u_i', velocity fluctuation components; i = 1, 2, 3; $e = \frac{1}{2}\sum_{i=1}^{3}\langle(u_i')^2\rangle$, turbulence intensity; $\varepsilon = \nu\sum_{i,j=1}^{3}\langle(\partial u_i'/\partial x_j)^2\rangle$, isotropic part of the total turbulent energy dissipation; $D = \frac{\nu}{2}\sum_{i,j=1}^{3}\langle(\partial u_i'/\partial x_j + \partial u_j'/\partial x_i)^2\rangle$, total dissipation; μ, dynamic viscosity; ν, kinematic viscosity; ρ, density; $-\rho\langle u'v'\rangle$, turbulent shear stress; U_∞, velocity at the edge of the boundary layer; Re_x, Reynolds number in the coordinate x; τ_w, shear stress at the streamline surface; $U_\tau = \sqrt{\tau_w/\rho_w}$, dynamic velocity; $U_* = U/U_\tau$, dimensionless velocity; $y_* = yU_\tau/\nu$, dimensionless coordinate; μ_T, turbulent transfer coefficient; R_T, turbulent Reynolds number; C_f, local friction coefficient; δ, boundary-layer thickness; f, correction function to the dissipation equation; C_1, C_2, C_μ, σ_ε, coefficients; $\vartheta = 100\sqrt{\langle(u')^2\rangle}/U_\infty$, amount of turbulence intensity of the incoming flow; Re, Reynolds number calculated over the tube diameter.

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DETERMINATION OF VAPOR BUBBLE BREAKAWAY DIMENSIONS IN HIGH-SPEED FLOWS

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UDC 536.423.1:532.517

A semiempirical relationship is proposed for determining the diameter at which vapor bubbles break away from a channel wall in high-speed flows of boiling liquids.

A large number of experimental and theoretical studies have been published to date concerning the determination of bubble breakaway diameter in liquid boiling on a heating surface under natural convection conditions. For example, a detailed review of the state of this problem can be found in [1, 2]. However, experimental studies under conditions of forced flow motion [3, 4] have produced qualitative and quantitative divergence from the results of the above studies. Thus, under certain conditions the bubble breakaway diameter is an order of magnitude or more lower than under corresponding natural convection conditions. The limited applicability of the few [3, 5] empirical descriptions is related to the insufficient volume of experimental data and the relatively narrow parameter ranges studied.

In connection with this fact, analytical studies are of definite interest. Unfortunately, to date the number of theoretical studies on determination of bubble breakaway diameter has been quite limited. The complexity of this problem is related foremost to the large number of factors affecting bubble breakaway conditions. Second, analysis of presently available studies, for example [6-9] et al., has shown that there is a diversity of opinion as to definition of the magnitude, direction, and character of the action of some forces. Moreover, in many studies the conditions used to define the moment of bubble breakaway from the wall are not well justified.

In considering the major factors affecting the breakaway of bubbles under conditions of both natural and forced convection the majority of authors agree that the main forces supporting bubbles during the breakaway process are forces produced by liquid relaxation in response to bubble growth and surface tension forces. Thus one can distinguish dynamic ($F_R \gg F_\sigma$) and quasistatic ($F_\sigma \gg F_R$) breakaway regimes according to [10].

Below we will consider the problem of vapor bubble breakaway in high-speed flows under quasistatic breakaway regime conditions at relatively low superheating levels. According to the results of [7] the force F_R may be neglected at $J_a < 10$. Low superheat levels define a relatively low value of vapor formation center density. Therefore the effect of bubble interaction on breakaway will not be considered.

It should be noted that at present there is not a generally accepted definition of the value of the force which compensates surface tension F_σ under such conditions. Thus, in [11, 12] the authors proposed a definition of the surface tension force at the moment of breakaway in the form

$$F_\sigma = 2\pi R_c \sigma, \tag{1}$$

where R_c is the radius of the microfissure which serves as the vapor formation center. The quantity R_c then corresponds to the radius of the critical vapor bubble nucleus.

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